Name: ________________

Chapter 4: Congruent Triangles

Guided Notes

Geometry
Fall Semester

4.1 Apply Triangle Sum Properties

Term	Definition	Example
triangle		
polygon		
sides		
vertices		

Classifying Triangles by Sides

scalene triangle		
isosceles triangle		
equilateral triangle		

Classifying Triangles by Angles

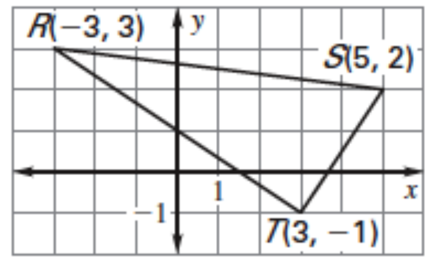
acute triangle		
obtuse triangle		
right triangle		
equiangular triangle		

interior angles		
exterior angles		
Theorem 4.1 Triangle Sum Theorem	The sum of the measures of the interior angles of a triangle is 180°.	
auxiliary lines		
Theorem 4.2 Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.	
corollary to a theorem		
Corollary to the Triangle Sum Theorem	The acute angles of a right triangle are complementary.	

Examples:

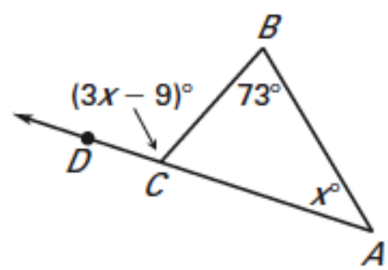
Use the diagram at right for examples 1 and 2.

1. Classify ΔRST by its sides. (Use the distance formula)

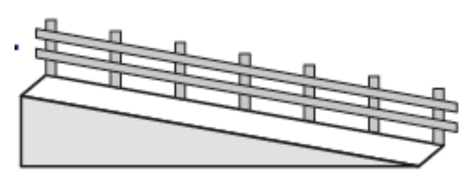


2. Determine if ΔRST is a right triangle.

3. Use the diagram at the right to find the measure of $\angle DCB$.



4. The front face of the wheelchair ramp shown forms a right angle. The measure of one acute angle is eight times the measure of the other. Find the measure of each acute angle.



4.2 Apply Congruence and Triangles

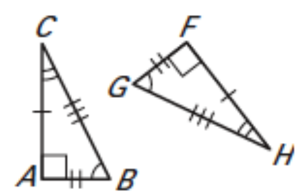
Term	Definition	Example
congruent figures		
corresponding parts		
Theorem 4.3 Third Angles Theorem	If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.	

Properties of Congruent Triangles

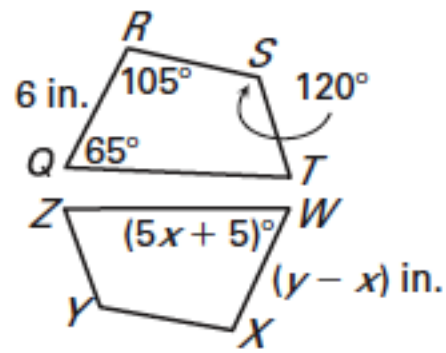
Theorem 4.4 Properties of Congruent Triangles		
Reflexive Property of Congruent Triangles	For any triangle *ABC*, $\Delta ABC \cong \Delta ABC$.	
Symmetric Property of Congruent Triangles	If $\Delta ABC \cong \Delta DEF$, then $\Delta DEF \cong \Delta ABC$.	
Transitive Property of Congruent Triangles	If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta JKL$, then $\Delta ABC \cong \Delta JKL$.	

Examples:

1. Write a congruence statement for the triangles. Identify all pairs of congruent, corresponding parts.



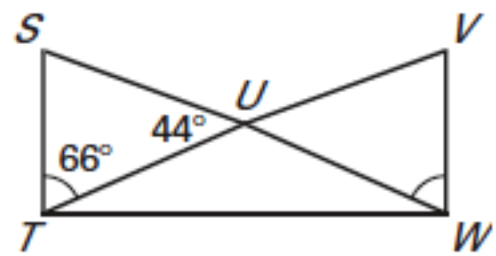
2. In the diagram, $QRST \cong WXYZ$.



a). Find the value of x.

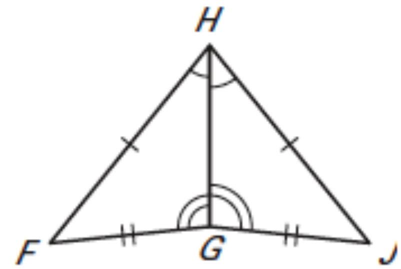
b). Find the value of y.

4. Find $m\angle V$.



5. **Given** $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$, $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$

Prove $\triangle FGH \cong \triangle JGH$



4.3 *Prove Triangles Congruent by SSS*

Term	Definition	Example
Postulate 19 Side-Side-Side (SSS) Congruence Postulate	If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.	

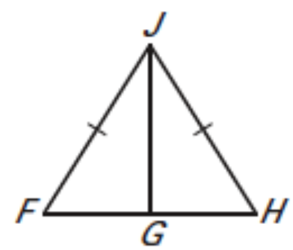
Examples:

1. Write a paragraph proof.

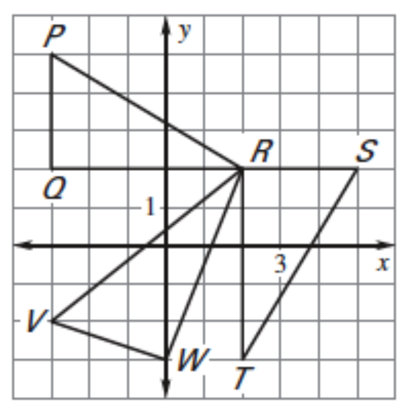
   Given: $\overline{FJ} \cong \overline{HJ}$

   Point G is the midpoint of $\overline{FH}$

   Prove: $\triangle FGJ \cong \triangle HGJ$



2. Determine whether $\triangle PQR$ is congruent to the other triangles shown at right.

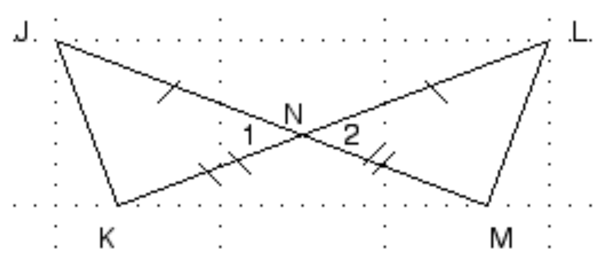


4.4 Prove Triangles Congruent by SAS and HL

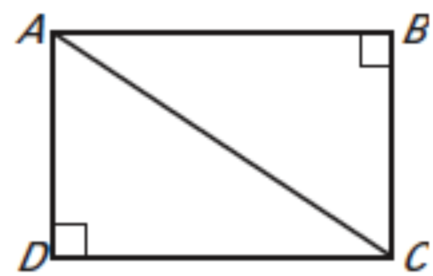
Term	Definition	Example
included angle		
Postulate 20 Side-Angle-Side (SAS) Congruence Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.	
right triangles	1. legs- 2. hypotenuse- 3. side opposite- 4. sides adjacent-	
Theorem 4.5 Hypotenuse-Leg (HL) Congruence Theorem	If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.	

Examples:

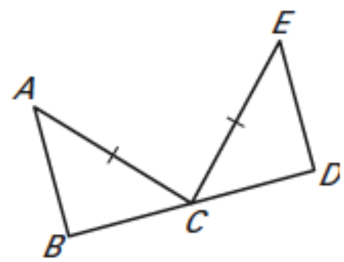
1. Given: $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$
   Prove: $\Delta JKN \cong \Delta LMN$



2. In the diagram $ABCD$ is a rectangle. What can you conclude about ΔABC and ΔCDA.

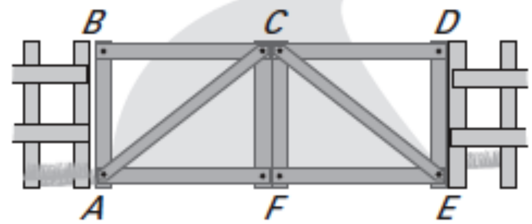


3. Given: $\overline{AC} \cong \overline{EC}$, $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$
$\overline{AC}$ is a bisector of $\overline{BD}$
Prove: $\Delta ABC \cong \Delta EDC$



Statements	Reasons
1. $\overline{AC} \cong \overline{EC}$	1. Given
2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. Given
3.	3. $\perp \rightarrow rt\angle s$
4.	4. $rt\angle s \rightarrow rt\Delta s$
5. $\overline{AC}$ is a bisector of $\overline{BD}$	5.
6.	6. Seg bisector $\rightarrow \cong segs$
7. $\Delta ABC \cong \Delta EDC$	7.

4. The entrance to a ranch has a rectangular gate as shown in the diagram. You know that $\triangle AFC \cong \triangle EFC$. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$?



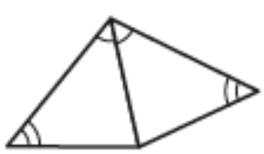
4.5 Prove Triangles Congruent by ASA and AAS

Term	Definition	Example
included side		
Postulate 21 Angle-Side-Angle (ASA) Congruence Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.	
Theorem 4.6 Angle-Angle-Side (AAS) Congruence Theorem	If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.	
flow proof		

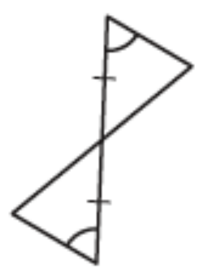
Examples:

1. Can the following triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

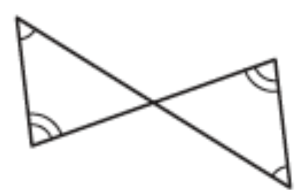
a).



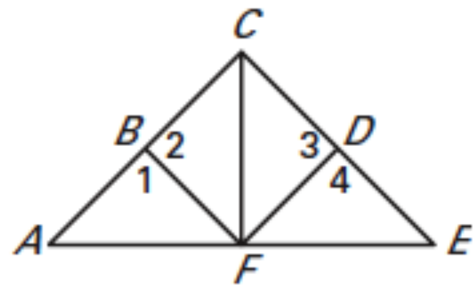
b)



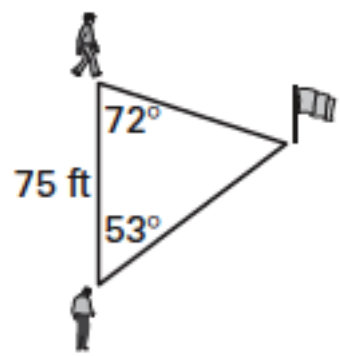
c)



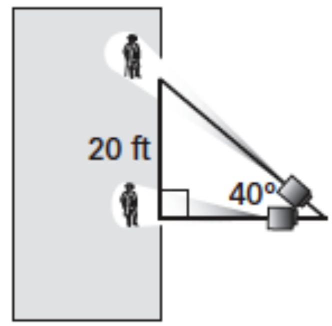
2. In the diagram, $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.



3. You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is $72°$, and the angle from your friend to the flag is $53°$. Is there enough information to locate the flag?



4. You are working two spotlights for a play. Two actors are standing apart from each other on the edge of the stage. The spotlights are located and pointed as shown in the diagram. Can one of the actors move without requiring the spotlight to move and without changing the distance between the other actor?



4.6 Use Congruent Triangles

Term	Definition	Example
congruent triangles		
Definition of Congruent Triangles (CPCTC)	Two triangles are congruent if and only if their corresponding parts are congruent. This is also known as the Corresponding Parts of Congruent Triangles are Congruent Theorem.	

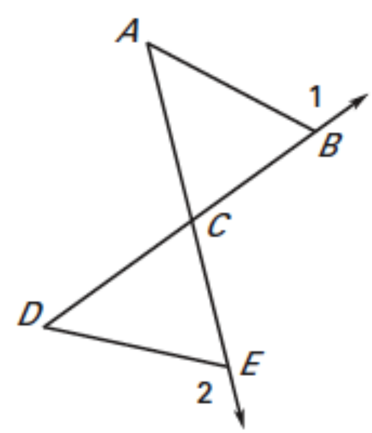
To show that a pair of corresponding parts of two triangles are congruent:

1. **Prove the two triangles are congruent.**
2. **Use the definition of congruent triangles (CPCTC) to show the corresponding parts are congruent.**

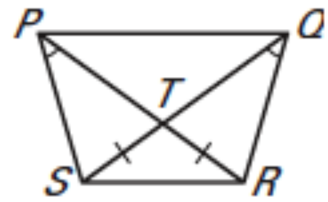
What can we say about SSA and AAA?	
SSA	SSA cannot be used as a proof of congruent triangles. See page 247.
AAA	AAA cannot be used as a proof of congruent triangles. AAA only proves the two triangles to be *similar*.

Examples:

1. Given: $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$
   Prove: $\overline{DC} \cong \overline{AC}$



2. Given: $\angle SPT \cong \angle RQT$, $\overline{TS} \cong \overline{TR}$
   Prove: $\overline{PR} \cong \overline{QS}$

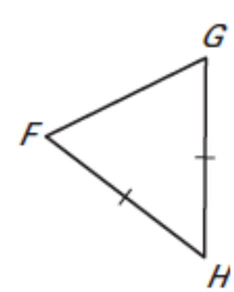


4.7 Use Isosceles and Equilateral Triangles

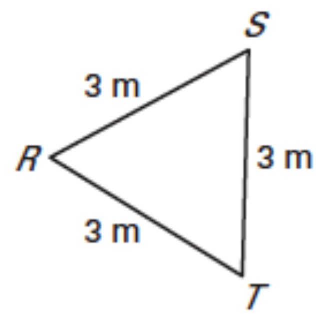
Term	Definition	Example
parts of an isosceles triangle	4 Vertex Angle— 5 Legs— 6 Base— 7 Base Angles—	
Theorem 4.7 Base Angles Theorem	If two sides of a triangle are congruent, then the angles opposite them are congruent.	
Theorem 4.8 Converse of Base Angles Theorem	If two angles of a triangle are congruent, then the sides opposite them are congruent.	
Corollary to the Base Angles Theorem	If a triangle is equilateral, then it is equiangular.	
Corollary to the Converse of Base Angles Theorem	If a triangle is equiangular, then it is equilateral.	

Examples:

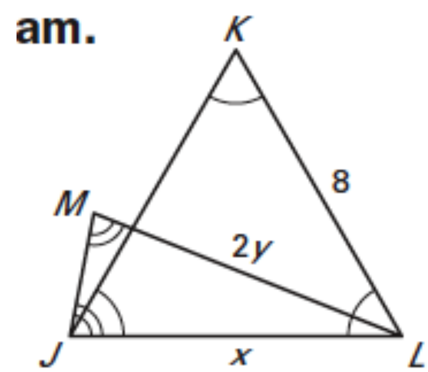
1. In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.



2. Find the measures of $\angle R$, $\angle S$, and $\angle T$.



3. Find the values of x and y in the diagram.



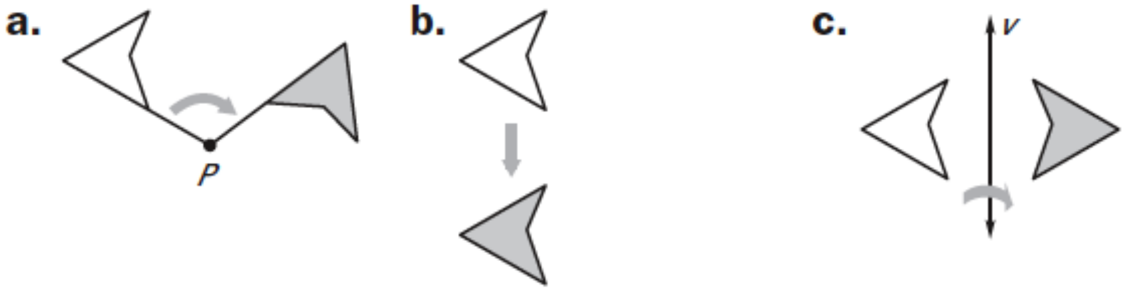
4.8 Perform Congruence Transformations

Term	Definition	Example
transformation		
image		
translation		
Coordinate Notation for a Translation		
reflection		
line of reflection		
Coordinate Notation for a Reflection in the x-axis		
Coordinate Notation for a Reflection in the y-axis		
Coordinate Notation for a Reflection in the line $y = x$		
rotation		

center (point) of rotation		
direction of rotation		1. **Clockwise** 2. **Counterclockwise**
angle of rotation		
congruence transformation		

Examples:

1. Name the type of transformation described in each picture.



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2. Figure $\boldsymbol{ABCD}$ has the vertices $\boldsymbol{A}(1,2), \boldsymbol{B}(3,3), \boldsymbol{C}(4,-1), \boldsymbol{D}(1,-2)$. Sketch $\boldsymbol{ABCD}$ and its image after the translation $(\boldsymbol{x},\boldsymbol{y}) \rightarrow (\boldsymbol{x}-4, \boldsymbol{y}+2)$.

$(\boldsymbol{x},\boldsymbol{y}) \rightarrow (\boldsymbol{x}-4, \boldsymbol{y}+2)$

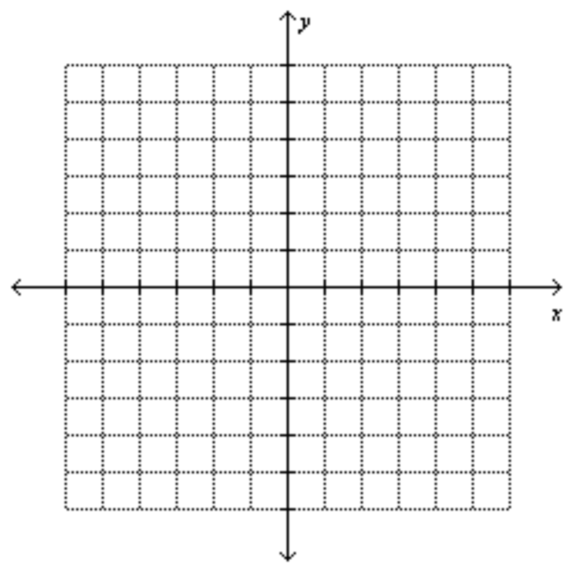
$\boldsymbol{A}(1,2) \rightarrow$ ________

$\boldsymbol{B}(3,3) \rightarrow$ ________

$\boldsymbol{C}(4,-1) \rightarrow$ ________

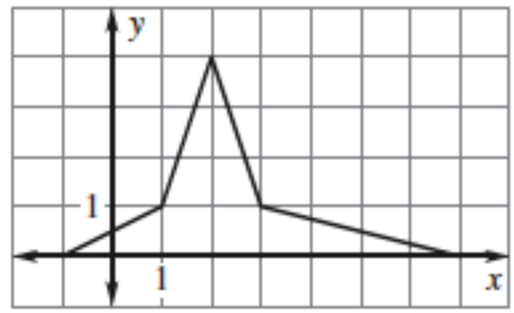
$\boldsymbol{D}(1,-2) \rightarrow$ ________

1.



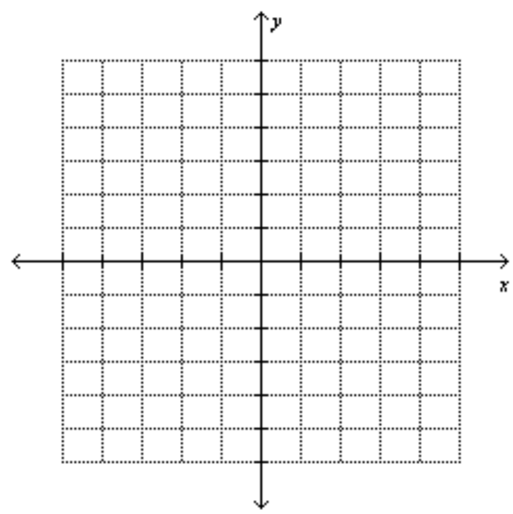
2. Reflect this figure in the x axis.

<u>Original Point</u> <u>Reflected Point</u>



3. Graph $\overline{JK}$ and $\overline{LM}$. Tell whether $\overline{LM}$ is a rotation of $\overline{JK}$ about the origin. If so, give the angle and direction of rotation.

   a. $J(3,1), K(1,4), L(-1,3), M(-4,1)$



b. $J(-2,1), K(-1,5), L(1,1), M(2,5)$